MATH 90 – CHAPTER 8 Name: .

8.1 Functions

Need To Know

- Idea of Functions
- Function Notation
- Functions and Graphs
- Applications

Review (after teaching):

- Definition = every x goes to only one y
- Do the "f" formula on the input of x
- Read: "f of x"
- f(x) = y

Function - Idea

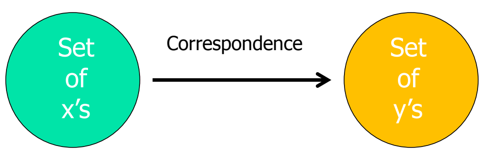
Consider: $y = 2x – 5$

Describe the characters of a solution to this equation?

The solution ___________.

They represent a ____________ between two things.

____



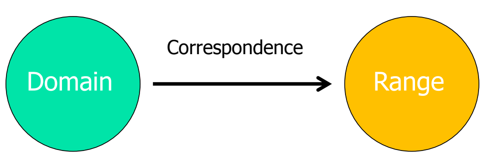
Function – Vocabulary

The ________ is the "input" set, often denoted by x's.

The _______ is the "output" set, often denoted by y's.

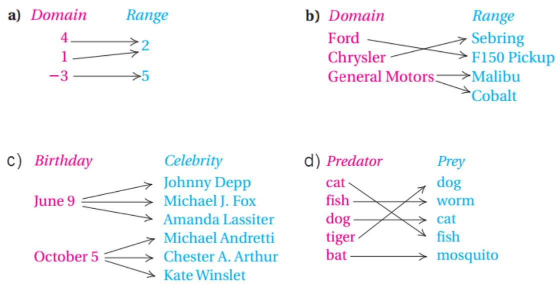
A _________ is a special correspondence (pairing) between two sets where every domain element is paired to exactly one range element.

(Every x pairs to exactly one y.)



Function – Examples (yes or no)

Every domain element pairs to exactly one range element.



Function – Examples (yes or no)

Every domain element pairs to exactly one range element.

Domain:	Bar codes in your grocery cart	The set of people in this class
Correspondence:	Cash register scanner	Ability to speak a language
Range:	Set of numbers for the price	Set of languages
Domain:	A set of rectangles	{ -2, -1, 0, 1, 2 }
Correspondence:	The area of each rectangle	The square of a number
Range:	A set of numbers	{ 0, 1, 4 }

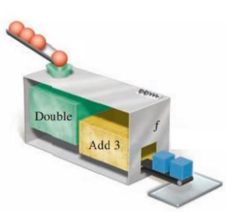
Function – Notation

A function is a correspondence which is determined by a rule or a formula or an equation.

Our notation for a function is **f (x)** said as "f of x".

Input

f(**x**) = 2**x** + 3



Find the function values:

f(5)

f(-7)

f(a)

If f(x) = 2x + 3, then f(5)= 13

If y = 2x + 3,
then y is 13 when x is 5.

Function – Practice

Function Facts:

- Think of "f" as
  a nick name for the formula
- ___________
- ___________________________
- f(x) has two meanings as a verb and a noun.
- Verb – f(x) says plug-in the "x" input value into the "f" formula.
- Noun – f(x) is the answer you get.
- x is _______________or input
- f(x) is ____________or output

Find the value of each function:

a) f(4) if $f(x) = 6x – 11$

b) g(-2) if $g(x) = -3x + 5$

c) g(a+1) if $g(x) = -3x + 5$

d) h(-5) if $h(n) = 2n^2 + 3n$

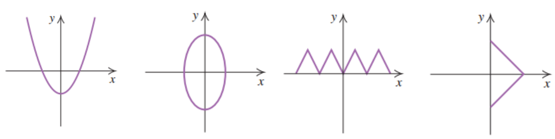
e) w(4) if $w(x) = \frac{x-3}{2x-5}$

Function – Graphs

Vertical Line Test

If it is possible for a vertical line pass through a graph more than once,
then the graph is not the graph of a function.

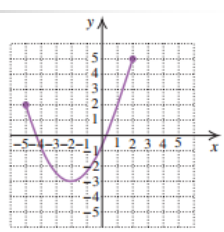
Examples

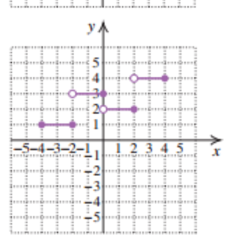


Function – Graphs

Function Graphs (for each graph)

1. Find f(1)
2. Any x-values where f(x) = 2
3. The domain of f
4. The range of f

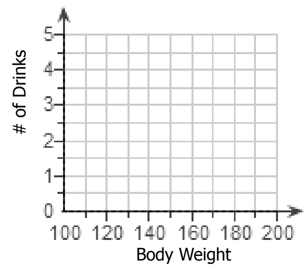




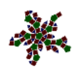
Function - Applications

Use the data to draw a graph,
Then determine the numbers of drinks to be considered intoxicated.
A) estimate for 140-lb person
B) predict for a 230-lb person

Input, Body Weight (in pounds)	Output, Number of Drinks
100	2.5
160	4
180	4.5
200	5



8.2 Functions – Domain and Range

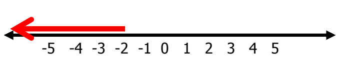


Need To Know

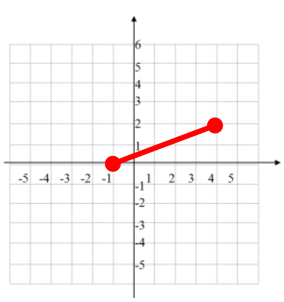
- Finding the Domain of a Functions
  1. From Ordered Pairs
  2. From Graphs
- Finding Domain Restriction for
  1. Rational Functions
  2. Polynomial Functions
  3. Physical Context
- Evaluating Piecewise Functions

Review: Set-Builder Notation 2.6

Set-builder Notation
Explains the set with a formula. {x | formula for x}



Write each set in set notation
1)



2) For the x's

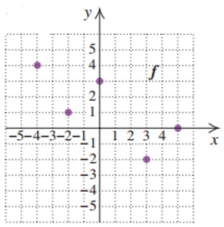
3) For the y's

Function – Domain & Range

The **domain** is the _______ set, often denoted by x's.
The **range** is the _________ set, often denoted by y's.

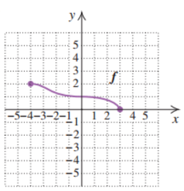
Find the domain and the range of each functions.

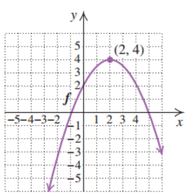
1) f = {(-2,5), (-1, 7), (0, 9), (5,6), (8, -3)}

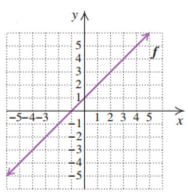
2) 

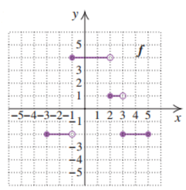
Function – Domain & Range

Find the domain and the range of each functions.









Domain Restrictions

When a function is given as an equation, the domain is not spelled out. It becomes our job to find the domain which is the set of all numbers that make the function "**work**".

One way is to ask yourself:

____

Determine the domain of:

$a)\ \ f(x) = 3x^2 - 4.$ $\qquad b)\ \ f(x) = \dfrac{2}{3x-4}.$

Domain Restrictions

When a function is given as an equation, the domain is not spelled out. It becomes our job to find the domain which is the set of all numbers that make the function "**work**".

One way is to ask yourself:
"Are there any x's for which f can not be computed?"

Determine the domain of:

$c)\ f(x) = |x-8|.$ $d)\ f(x) = \dfrac{6x-7}{x^2-25}.$

Domain Restrictions - Context

The record R for the 400-m run "t" years after 1930 is given by R(t) = 46.8 – 0.075t.
What's the domain of the function?

The height h, in feet, of a fireworks display, t seconds after having been launched from an 80-ft high rooftop, is given by $h(t) = -16t^2 + 64t + 80$
What's the domain of the function?

Piecewise Defined Functions

____________________are described by different equations for various parts of the domain.

1) $f(x) = \begin{cases} 3x, & \text{if } x < 4 \\ x+2, & \text{if } x \geq 4 \end{cases}$

Find:
f(-3)
f(0)
f(6)

2) $g(x) = \begin{cases} x+3, & \text{if } x \leq -3 \\ x^2, & \text{if } -3 < x \leq 4 \\ 4x, & \text{if } x > 4 \end{cases}$

Find:
g(6)
g(-3)
g(0)

8.3 Graphing Functions

Need To Know

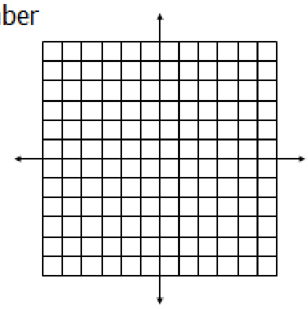
- Review Linear Equation
- Linear Functions
  1. Graphing
  2. Finding Domain and Range
- Nonlinear Functions
  1. Finding Domain and Range
  2. Graphing
- Translating Functions

Recall – Linear Equations

Equations of Lines

1. Standard Form: $Ax + By = C$
2. Slope-Intercept Form: $y = mx + b$
3. Point-Slope Form: $y - y_1 = m(x - x_1)$
4. Horizontal Line: y = number
5. Vertical Line: x = number

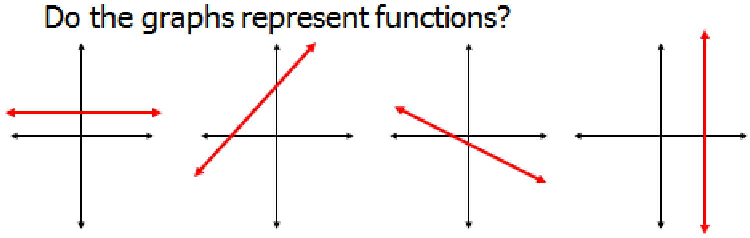
Graph the equation
$2x + 3y = 6$



Line Graphs and Functions

Recall: Vertical Line Test

Do the graphs represent functions?



Linear Functions:

1. ______________________________
2. $f(x) = b$ is a constant function horizontally through $(0,b)$.

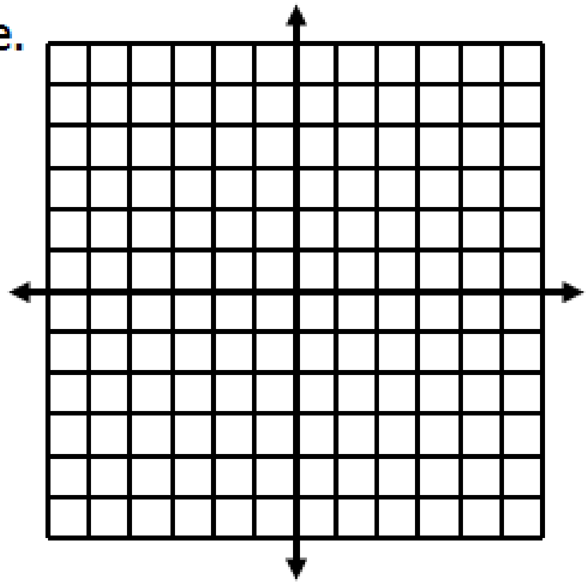
Graphing Linear Functions

Graph each and give the domain and range.

$f(x) = -\frac{2}{5}x + 4$

$g(x) = 4x - 6$

$h(x) = 5$



Nonlinear Functions

Extend to add domain and graph

Function Families	Parent Function	Children Functions
A. Linear:	$f(x) = x$	
B. Quadratic:	$f(x) = x^2$	
C. Polynomial:	$f(x) = ax^n + dx^m + \ldots$	
D. Absolute Value:	$f(x) = \lvert x \rvert$	
E. Rational:	$f(x) = \frac{1}{x}$	

Nonlinear functions create graphs that are not straight lines.

Classify each function below as one of the five types above.

(1) $f(x) = -\frac{2}{5}x + 4$ (2) $g(x) = \frac{x-4}{3x+8}$ (3) $h(x) = \lvert 3x - 7 \rvert$

(4) $t(x) = 3x^2 + 8x - 4$ (5) $s(p) = 4p^5 - 1.6$ (6) $g(t) = 657 - 0.2t$

Nonlinear Functions

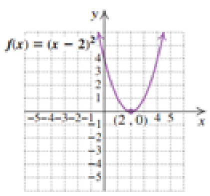
Find the domain of each function:

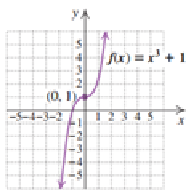
(1) $f(x) = -\frac{2}{5}x + 4$ (2) $g(x) = \frac{x-4}{3x+8}$ (3) $h(x) = |3x - 7|$

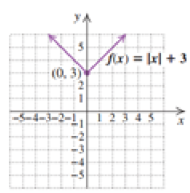
(4) $t(x) = 3x^2 + 8x - 4$ (5) $s(p) = 4p^5 - 1.6$ (6) $g(t) = 657 - 0.2t$

Nonlinear Functions

Find the range of each function:

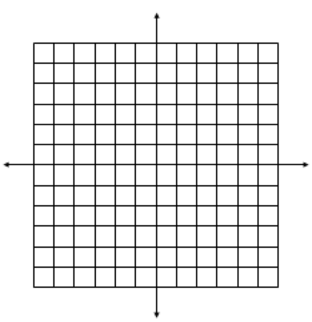






Graphing Nonlinear Functions

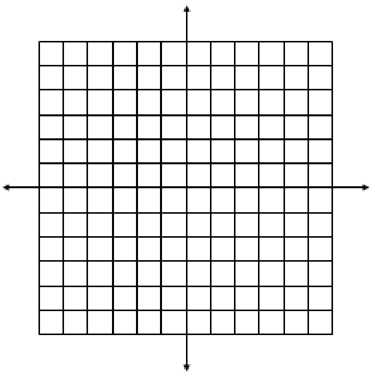
Lines are best graphed with 3 points.
Nonlinear functions need at least 5 to 7 points to see the shape



Graph:

f(x) = |x| - 5

Graphing Nonlinear Functions



Graph:

1) $f(x) = |x + 2|$

2) $f(x) = -x^2 + 1$

Translating & Application of Functions

Lauren had her hair cut to a length of 5 inches in order to donate the hair to Locks of Love. Her hair then grew at a rate of inch per month. Formulate a linear function to model the length ***L(t)*** of Lauren's hair ***t*** months after she had the haircut, and determine when her hair will be 15 inches long.

Translating & Application of Functions

As demand has grown, worldwide production of small cars rose from 14.5 million in 2002 to 19 million in 2007. Let a(t) represent the number of small cars produced t years after 2000

a) Find a linear function that fits the data.
b) Predict the number of cars produced in 2013.
c) In what year will 35? million cars be produced?

8.4 Graphing Functions

Need To Know

- Operations on Functions
  1. Add
  2. Subtract
  3. Multiply
  4. Divide
- Domains and Graphs

Operations of Functions

The Algebra of Functions

If *f* and *g* are functions and *x* is in the domain of both functions, then:

1. $(f+g)(x) =$
2. $(f-g)(x) =$
3. $(f \cdot g)(x) =$
4. $(f/g)(x) =$

Practice – Function Operations

For $f(x) = 3x - x^2$ and $g(x) = 2x + 1$, find:

a) $(f+g)(4)$

b) $(f \cdot g)(-1)$

c) $(f/g)(x)$

d) $(f-g)(x)$

Domain of Function Combinations

To obtain the domain for an operations of function

1) Find the domain of f
2) Find the domain of g
3) Make sure common elements in both the domains of f and g also work in the final combination (operation) of f and g. (Usually, this is only tricky with division.)

Practice Finding Domains

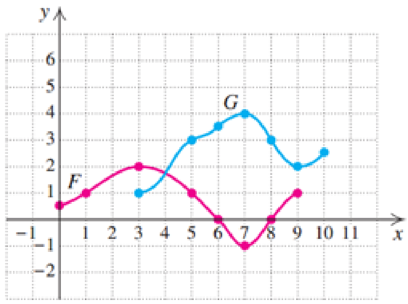
Given $f(x)=\dfrac{2}{x+1}$ and $g(x)=x-3$, find domains of $(f+g)(x)$, $(f-g)(x)$, and $(f\cdot g)(x)$.

Practice Finding Domains

Given $f(x)=\dfrac{2}{x+1}$ and $g(x)=x-3$, find domains of

$$(f/g)(x)=\frac{f(x)}{g(x)}$$

Operations and Domains by Graphs



1. Determine each
   (F + G)(5) =
   (F · G)(6) =
   (G – F)(7) =

2. Find domain of
   F
   G
   F – G
   G / F

3. Graph
   F + G

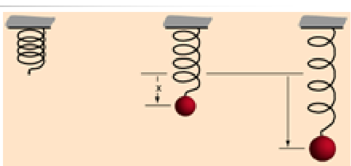
8.5 Formulas and Variation

Need To Know

- Variation
  1. Direct
  2. Inverse
  3. Joint

Direct Variation

As the mass(m) gets larger the amount of stretch on the spring (x) gets larger. The stretch ***varies directly*** with the mass.



When we say that there is *direct variation*, that ______________________________, or that *y is proportional to x* then

it's ______________________________,
where *k* is a nonzero constant.

(The number *k* is called the *constant of proportionality*.)

Direct Variation

Translating Variation:

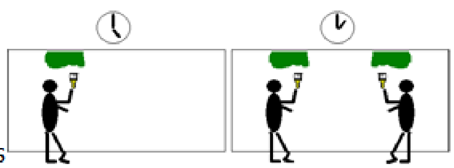
A. Find the equation **TYPE** (without numbers).
B. Find the ***k***.
C. Write the specific equation (plug in ***k***).
D. Answer follow up question

Example:

Find the variation constant and an equation of variation if *y* varies directly as *x*, and *y* = 15 when *x* = 3.

Inverse Variation

Suppose it takes one person 8 hours to paint a building.
Two people take 4 hours and 4 people take 2 hours



The time ***varies inversely*** with the number of people.

When we say that there is *inverse variation*, that is ______________________________, or that *y is inversely proportional to x* then

it's ______________________________

(The number *k* is called the *constant of proportionality.*)

Inverse Variation

Translating Variation:

A. Find the equation **TYPE** (without numbers).
B. Find the ***k***.
C. Write the specific equation (plug in ***k***).
D. Answer follow up question.

The time, *t*, required to empty a tank varies inversely as the rate, *r*, of pumping. If a pump can empty a tank in 90 minutes at the rate of 1080 kL/min, how long will it take the pump to empty the same tank at the rate of 1500 kL/min?

Joint and Combined Variation

When a variable varies directly with more than one other variable, we say that there is *joint variation.*

> **Joint Variation**
>
> *y* varies *jointly* as *x* and *z* if, for some nonzero constant *k*, $y = kxz$.

For example, in the formula for the volume of a right circular cylinder, we say that *V* varies *jointly* as *h* and the square of *r*.

So V = ________________________

Joint Variation

Translating Variation:

A. Find the equation **TYPE** (without numbers).
B. Find the ***k***.
C. Write the specific equation (plug in ***k***).
D. Answer follow up question

Find an equation of variation if *a* varies jointly as *b* and *c*, and *a* = 48 when *b* = 4 and *c* = 2.

Variation Application

Translating Variation:

A. Find the equation **TYPE** (without numbers).
B. Find the ***k***.
C. Write the specific equation (plug in ***k***).
D. Answer follow up question.

The time that it takes to download a movie file varies inversely as the transfer speed of the internet connection.
A typical full-length movie file will transfer in 48 min at a transfer speed of 256 KB/s.
How long will it take to transfer the same movie file at a transfer speed 32 KB/s?